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Form Approved  
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1. REPORT DATE (DD-MM-YYYY) 02/26/2015		2. REPORT TYPE Final Technical Report		3. DATES COVERED (From - To) 01/01/2014 - 31/12/2014	
4. TITLE AND SUBTITLE Particle Methods for Atmosphere and Ocean Modeling				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER N00014-14-1-0075	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Krasny, Robert Department of Mathematics University of Michigan Ann Arbor, MI 48109-1043 (734)-763-3505 krasny@umich.edu				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) The Regents of the University of Michigan Division of Research Development Administration 3003 South State Street 1058 Wolverine Tower Ann Arbor, MI 48109-1274				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Naval Research 875 North Randolph Street Arlington, VA 22203-1995				10. SPONSOR/MONITOR'S ACRONYM(S) ONR	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for Public Release; distribution is Unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The award supported Peter Bosler as a postdoc in the Department of Mathematics at the University of Michigan. The project developed a novel Lagrangian particle method (LPM) for geophysical fluid flow simulations on a rotating sphere. The method is potentially relevant to Naval operations that rely on accurate and efficient modeling of atmosphere and ocean dynamics. We developed new remeshing and refinement schemes to overcome the loss of accuracy due to distortion in the particle distribution. The results were disseminated through published articles and presentations at conferences and seminars.					
15. SUBJECT TERMS Lagrangian particle method, barotropic vorticity equation, sphere, adaptive refinement and remeshing, atmosphere and ocean modeling					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Robert Krasny
U	U	U	UU	2	19b. TELEPHONE NUMBER (Include area code) (734)-763-3505

Standard Form 298 (Rev. 6/90)  
Prescribed by ANSI Std. Z39.18

20150309002

# Final Technical Report

ONR Award Number: N00014-14-1-0075

Title: Particle Methods for Atmosphere and Ocean Modeling

PI: Robert Krasny, University of Michigan

## Summary

This award supported Peter Bosler as a postdoc in the Department of Mathematics at the University of Michigan in the Fall 2013 and Winter 2014 semesters. The award provided 50% support and the remaining 50% support came from a teaching appointment. Dr. Bosler completed his thesis in May 2013 under the co-supervision of Professor Robert Krasny in the Mathematics Department and Professor Christiane Jablonowski in the Department of Atmospheric, Oceanic and Space Sciences at the University of Michigan [1]. In August 2014 Dr. Bosler started employment at Sandia National Laboratories, Albuquerque, where he was awarded the 2014 John von Neumann Postdoctoral Research Fellowship in Computational Science.

This project develops a novel Lagrangian particle method (LPM) for geophysical fluid flow simulations. The goal is to develop significantly more accurate and efficient algorithms for simulation of atmosphere and ocean dynamics.

The algorithms currently used in geophysical fluid flow simulations are based mainly on Eulerian and semi-Lagrangian mesh-based schemes, and the numerical results often exhibit diffusive and dispersive errors that arise from the explicit discretization of the advection terms in the equations of motion. In contrast to this, LPM represents the flow map using Lagrangian particles that carry vorticity and the associated velocity field is obtained from the Biot-Savart integral. Moreover we use fourth-order Runge-Kutta time stepping to compute the particle trajectories with high accuracy.

One difficulty with the Lagrangian approach is the distortion of the particle distribution in time, but we developed new remeshing and refinement schemes to overcome this problem. In this way LPM avoids the errors commonly seen with mesh-based schemes and this is demonstrated in the articles cited below.

We applied LPM to solve the barotropic vorticity equation on a rotating sphere and showed how the scheme is able to resolve small-scale features in the flow [2]. Movies can be viewed on Dr. Bosler's website [3]. In related work we applied LPM to compute tracer transport on the sphere and obtained results that compare favorably with a variety of other methods in terms of tracer error and preservation of nonlinear tracer correlations [4]. A longer article on this topic is in preparation in which we compare LPM with the well-known Lin-Rood scheme [5]. Another application of LPM to the stability of the polar vortex is in preparation [6].

A key goal for us is to extend LPM to solve the shallow water equations. In this case the particles carry height and divergence in addition to vorticity. This approach requires solving two Poisson equations, one for the vorticity and stream function, and another for the divergence and velocity potential function. We solve these Poisson equations by convolution with the Green's function and use adaptive quadrature to evaluate the resulting singular integrals. Preliminary results are promising and we believe this goal is within reach.

## Presentations

- California Institute of Technology, Pasadena, January 9, 2014, seminar (Bosler)
- Sandia National Laboratories, Albuquerque, February 5, 2014, seminar (Bosler)
- Conference on “PDEs on the Sphere”, National Center for Atmospheric Research, Boulder, April 7–14, 2014, contributed talk (Bosler)
- Math Department, University of California, San Diego, June 5, 2014, colloquium (Krasny)
- SIAM Annual Meeting, Chicago, July 7–11, 2014, contributed talk (Bosler)
- Department of Mechanical Engineering and Science, Kyoto University, Kyoto, October 20, 2014, seminar (Krasny)
- Department of Applied Analysis and Complex Dynamical Systems, Kyoto University, Kyoto, October 27, 2014, seminar (Krasny)
- Conference on “Numerical Methods and Analysis for Structures and Singularities in Fluids”, Nagoya University, Nagoya, December 8–9, 2014, invited lecture (Krasny)
- American Geophysical Union Fall Meeting, San Francisco, December 15–19, 2014, contributed talk (Bosler)

## References

- [1] P.A. Bosler (2013) Particle Methods for Geophysical Flow on the Sphere, Ph.D. Thesis, University of Michigan
- [2] P. Bosler, L. Wang, C. Jablonowski, R. Krasny (2014) A Lagrangian particle/panel method for the barotropic vorticity equations on a rotating sphere, *Fluid Dynamics Research* 46, 031406
- [3] P. Bosler, <http://www-personal.umich.edu/~pbosler/research/sphere-movies>
- [4] P.H. Lauritzen, P.A. Ullrich, C. Jablonowski, P.A. Bosler, D. Calhoun, A.J. Conley, T. Enomoto, L. Dong, S. Dubey, O. Guba, A.B. Hansen, E. Kaas, J. Kent, J.-F. Lamarque, M.J. Prather, D. Reinert, V.V. Shashkin, W.C. Skamarock, B. Sorensen, M.A. Taylor, M.A. Tolstykh (2014) A standard test suite for two-dimensional linear transport on the sphere: results from a collection of state-of-the-art schemes, *Geoscientific Model Development*, 7, 105–145
- [5] P. Bosler, J. Kent, C. Jablonowski, R. Krasny, A Lagrangian particle method for tracer transport on the sphere, in preparation
- [6] P. Bosler, C. Jablonowski, R. Krasny, Simulation of polar vortex dynamics due to sudden stratospheric warming, in preparation